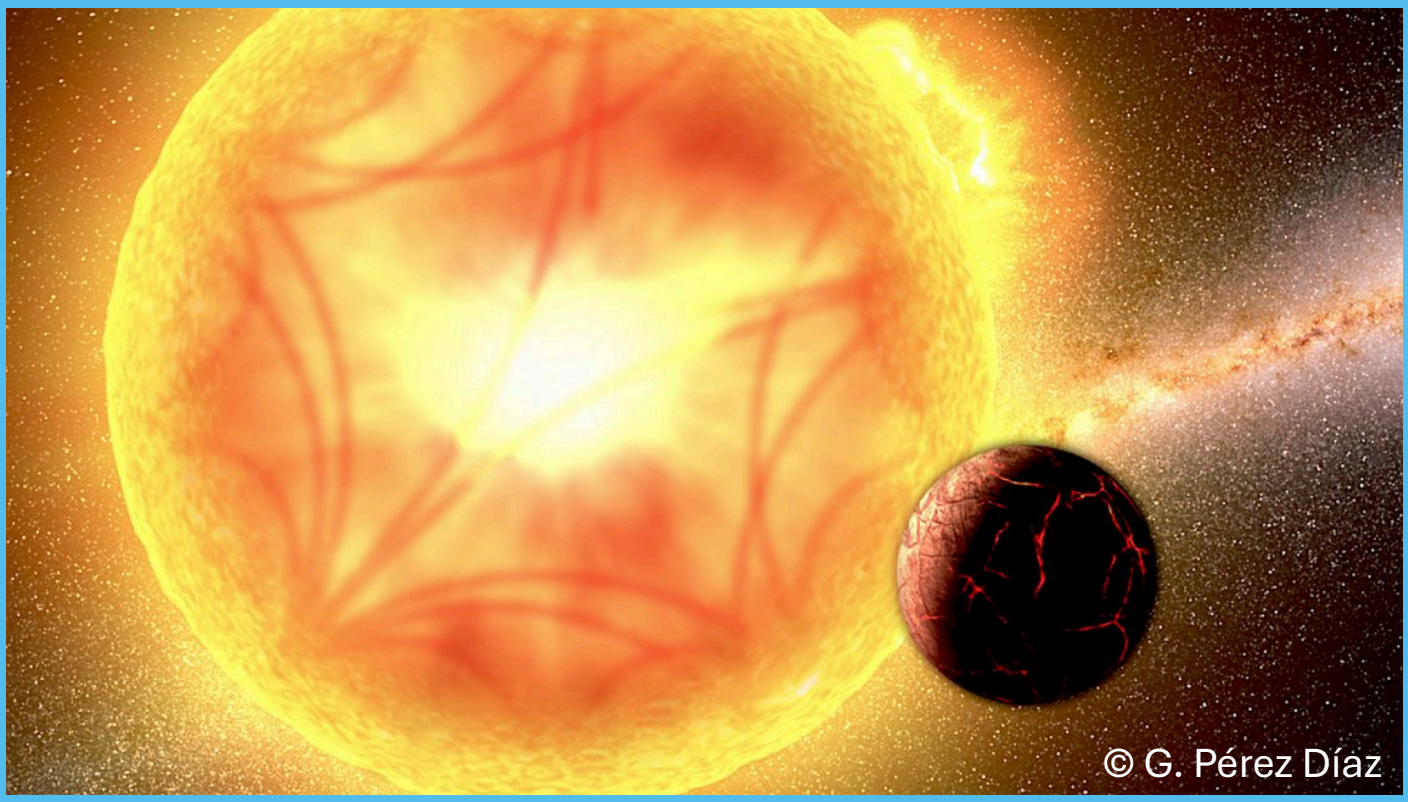
KU LEUVEN Junior College

4D-STAR

Asteroseismology & Exoplanets in the Milky Way



Conny Aerts, Kaustubh Hakim, Mieke De Cock & Rose Stanley

Content

Preface 2
About Junior College STEM 2
Asteroseismology and Exoplanets in the Milky Way 2
Practical Information 4
1. Some Background 5
1.1. Prerequisite Knowledge 5
1.1.1. Waves 5
1.1.2. Spherical Coordinates 8
1.2. Advanced Material (Optional) 10
1.2.1. Waves on a String 10
1.2.2. Waves on a Sphere 12
1.2.3. Fourier Analysis 14
2. Exercises for Chapter 1 16
3. Project: Let's Make You a Real Asteroseismologist! 20
4. Exercise for Chapter 4 23
Appendix 1: Physical and Astronomical Constants 25
Appendix 2: Effective Temperatures of Stars 26

Preface

About Junior College STEM

Smartphones, computers, the internet ... It's hard to imagine a world without them. All these technologies are the result of scientific research and technical innovations. Science and technology are the driving forces behind innovation in diverse areas such as healthcare, communications, transportation, food, and the environment. Think of the development of new ways to utilise renewable energy sources, the creation of mathematical models to predict and control the spread of diseases, research into ways to use natural resources more efficiently, or the enormous potential of quantum computers, and so on. As a scientist, you can therefore have a direct impact on the world around you: you contribute to improving the quality of life and solving major societal challenges.

In a scientific course, you also learn the skills of critical thinking and problem-solving. You learn to approach problems systematically, find solutions using logic and scientific and mathematical methods, and critically evaluate information. These skills are valuable in all kinds of professions, not just in traditional STEM disciplines. The demand for professionals with STEM skills is therefore high and continues to grow. These skills are also useful in your daily life: they help you make better-informed decisions and give you the tools to quickly and effectively understand new information.

With Junior College STEM, we want to give you a taste of what a university science degree can entail. In our modules, you'll learn to approach and solve complex, challenging problems scientifically. We want to help you choose your higher education course: which courses would suit you (and perhaps which wouldn't)?

Asteroseismology and Exoplanets in the Milky Way

In this Junior College module, you will be introduced to asteroseismology. **Asteroseismology** is a young, rapidly expanding field of science, in which Belgian astronomers are playing a pioneering role. For example, the Kavli Prize in Astrophysics was awarded to asteroseismologist Professor Conny Aerts (KU Leuven), the first and so far only Belgian to receive this prestigious prize in 2022:
https://www.kavliprize.org/category/astrophysics.

How can we use the light from stars to determine their internal physics and chemistry, as well as their ages? How does this knowledge of stars also provide information about possible life forms on their exoplanets? By the end of this module, you will be able to determine the mass, radius, and age of stars from modern satellite data of starquakes based on project work. You will also be able to use simple physics concepts to deduce whether exoplanets are rocky or gaseous and whether they lie in the habitable zone of their host star.

The *first chapter* provides an introduction to stars and their properties. You'll learn how astronomers study the life cycle of stars. You'll learn to calculate the key timescales in a star's life and how nuclear fusion in the star's interior changes its chemical composition over the course of its life, and then releases the created materials into the Milky Way at the end of its life. From this material, new generations of stars are born.

Chapter 2 is an introduction to asteroseismology, a relatively new branch of astronomy that uses observations of starquakes to derive the properties of stars with unprecedented precision. You'll learn to work with the frequencies of the sound waves generated by starquakes to determine the star's radius and mass. You'll also see how these frequencies can be derived from a Fourier transform of the star's brightness measurements.

Chapter 3 delves into more complex aspects, particularly the influence of the Coriolis force, which cannot be neglected in stars that are more massive and hotter than the Sun. You will learn how the internal rotation of stars can be inferred from shifts in their starquake frequencies. Consequently, you will be confronted with the limitation of the dominant basic assumption in astronomy: that stars are perfect spheres, while in reality they are flattened by centrifugal force. We conclude Chapter 3 with new directions in computational astronomy, where this oversimplified assumption of spherical symmetry is discarded.

Chapter 4 focuses on exoplanets and their relationship to their host stars, with the Earth-Sun system serving as the most prominent example. We explain how to efficiently find exoplanets and their basic properties, such as mass, radius, average density, and chemical composition. The distance to their host star is also important, because together with the star's surface temperature, it determines whether or not the exoplanet lies within the star's habitable zone. To derive these planetary properties, we need to know the star's mass, radius, and surface temperature. This knowledge is provided by the asteroseismological study of the host star. This provides an insight into how asteroseismology and exoplanet studies go hand in hand.

Practical Information

This module consists of a series of **videos** in which "real" KU Leuven astronomers provide the necessary theoretical knowledge. After watching these videos, you can get started with the **exercises** in this booklet. The exercises will familiarise you with the three main timescales in the life of stars (Part 2) and the density and habitability of exoplanets (Part 4). In the **project work** (Part 3), you will be assigned a "personal star", the starquake frequencies of which will be made available electronically. Just like a true asteroseismologist, you will use these data to determine the radius, mass, and age of "your" star. Your teacher will give you more information about how this module will be implemented in class.

This module is based on an integrated STEM approach, interweaving elements of mathematics, physics, chemistry, technology, and computer science. Such an iSTEM approach is typical of modern astronomy research. The module builds on your **prior knowledge** from these subjects. At the beginning of this text (Part 1), we review some basic concepts of waves and spherical coordinates.

1. Some Background

1.1. Prerequisite Knowledge

1.1.1. Waves

In the most general case, a **wave** is a disturbance from equilibrium that propagates through some medium, such as gas, water or a string. In the simplest case, a wave takes the form of regularly spaced alternating peaks and troughs in the medium **(Fig. 1.1)**. This wave is a simple sinusoid travelling at a constant velocity.

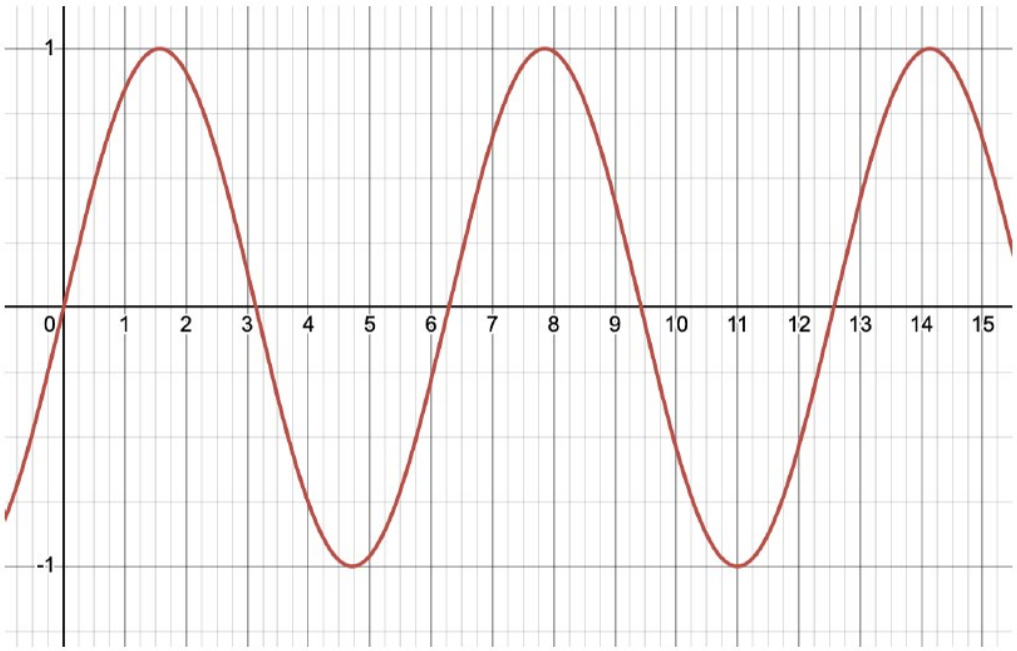


Figure 1.1. A simple sinusoid wave travelling at a constant velocity

When you add up several (or infinitely many) sinusoidal waves, you can get some quite complex-looking behaviour **(Fig. 1.2)**. The phenomenon in which waves overlap and add up or cancel out is called **interference**. In contrast to waves, everyday physical objects do not exhibit interference. For instance, if you bump into someone else, you don't briefly form a person twice as large, before continuing on your separate ways; rather, you both stop and then maneuver around each other, because you can't both occupy the same

spot simultaneously. Waves are different. Waves have no trouble passing through each other, and when they do, they interfere and form a resulting wave that is simply the sum of the individual waves. In **Figure 1.3** you can see two waves, in red and blue, and the sum of those waves, in black.

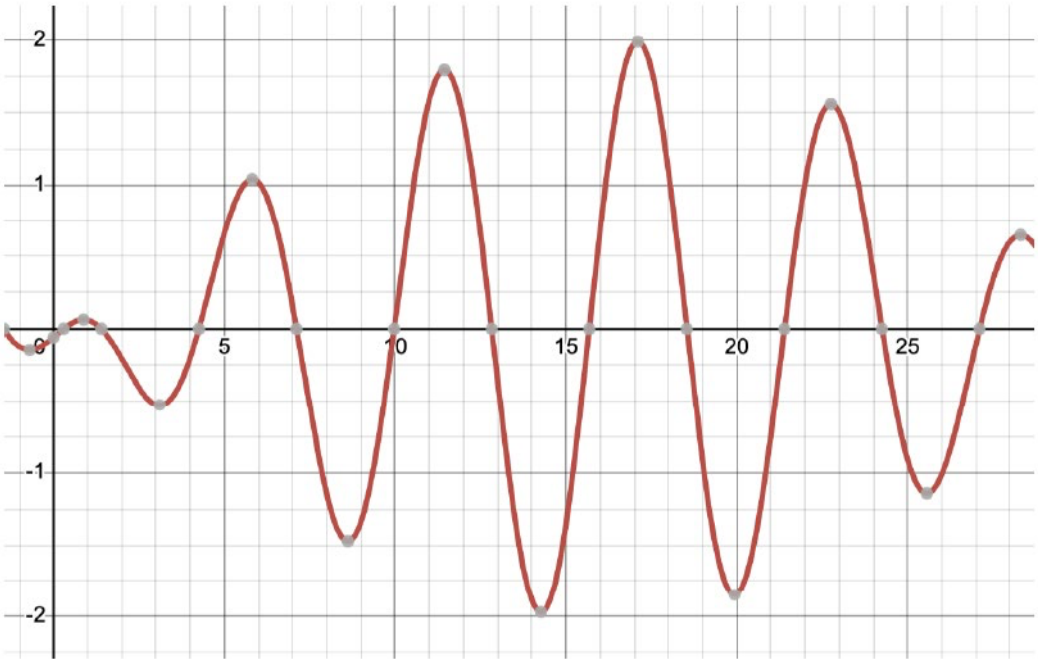


Figure 1.2. A complex wave created by the addition of several simple sinusoids

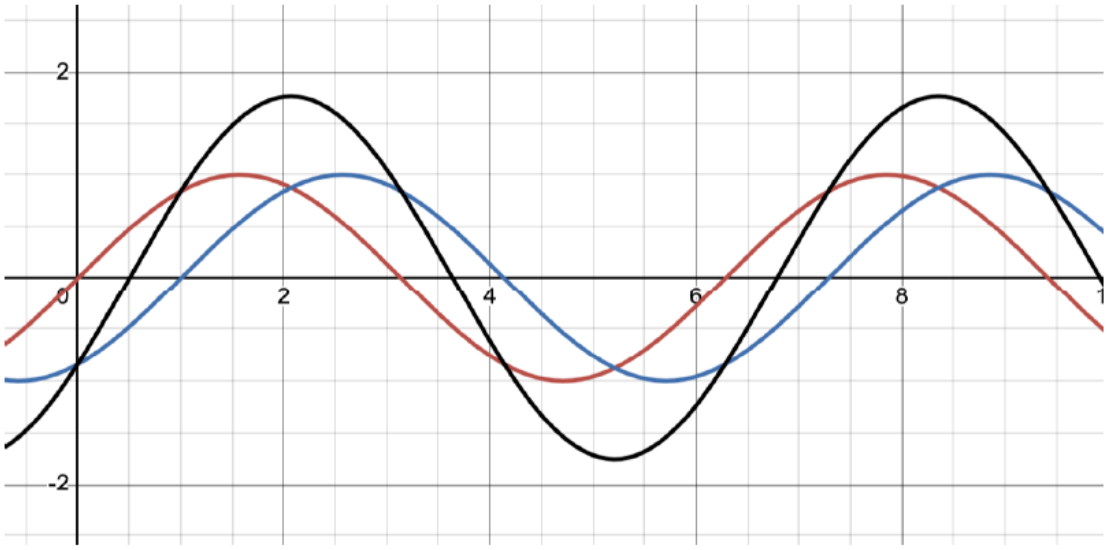


Figure 1.3. A demonstration of interference in waves. The red and blue lines are simple sinusoids. At each point along the line, the red and blue lines can be added, resulting in the black line. You can verify this by measuring the distance of the red and blue line from the horizontal axis at a single point, adding these numbers up and comparing this to the distance of the black line from the horizontal axis at the same point.

Perhaps surprisingly, all waves can be written as the sum of travelling sinusoidal waves. The complex-looking waves shown in Figure 1.2 are the sums of some number, perhaps infinitely many, of sinusoidal waves.

Now for some terminology. For a travelling sinusoidal wave, we have the following terms:

- **Wavelength**: the distance between two consecutive peaks on the wave, typically denoted by λ **(Fig. 1.4)**. All three of the black double-ended arrows in Figure 1.4 represent one wavelength of the wave.

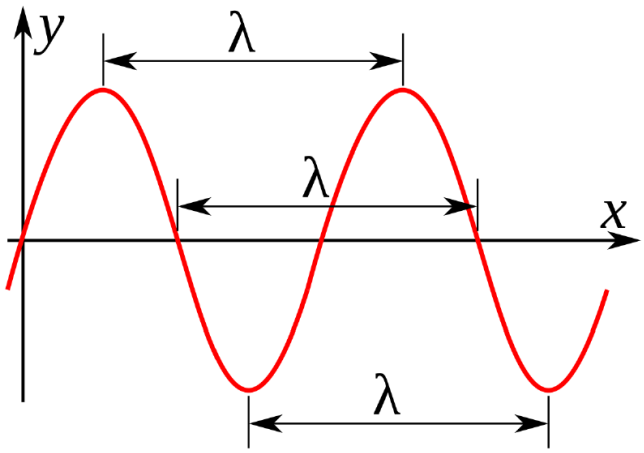


Figure 1.4. A simple sinusoid wave with three black arrows, each denoting the wavelength of the wave, λ

- **Frequency**: The number of wavelengths that the wave travels per second, typically denoted by f.
- **Period**: The reciprocal of the frequency, or the amount of time it takes the wave to travel one wavelength, typically denoted by T. Mathematically, $f = \frac{1}{T}$.
- **Amplitude**: The maximum distance of the peak or trough of the wave from the 0 point, represented by the x-axis in **Figure 1.5**. Both of the black double-ended arrows in Figure 1.5 represent the amplitude of the wave.
- **Speed of sound** (in a medium): The velocity of a wave in a medium, typically denoted by v. This depends solely on the medium and, perhaps surprisingly, not at all on the wave's wavelength or frequency.

If you know any two of the wavelength, frequency, and the speed of sound, then you can calculate the third through the equation $v = f\lambda$.

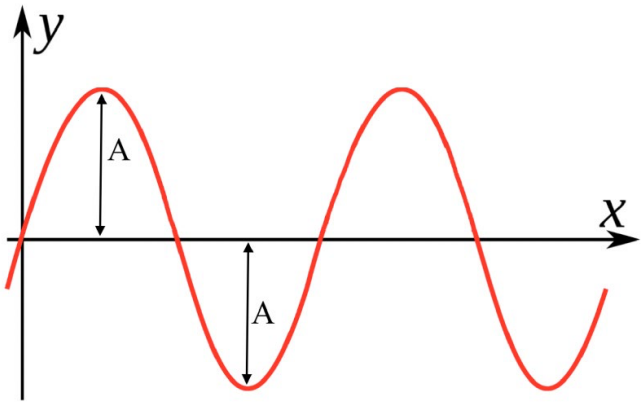


Figure 5. A simple sinusoid wave with two black arrows, each denoting the amplitude of the wave, A

For a one-dimensional travelling sinusoidal wave with amplitude A, the equation of the displacement y from equilibrium $y = 0$ as a function of position x and time t is:

$$y(x,t) = A \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

1. *Using the terms defined above and their mathematical equivalents, substitute these in until you can convince yourself that this wave travels at speed* $v = f\lambda$.

1.1.2. Spherical Coordinates

You're probably familiar with the Cartesian coordinate system, where a point is represented by its positions along the perpendicular axes. In three dimensions, a point would be described by three coordinates (x, y, z), its positions along each of the three axes. There are other coordinate systems as well. For describing phenomena in stars, which are spherical, we will use **spherical coordinates**. In a spherical coordinate system, a point would be described by the following three attributes **(Fig. 1.6)**:

- r : the **radial component**. This is the distance of the point from the origin.
- θ: the **azimuthal component**. This is the angle between the point and the z-axis, and it ranges from 0 to π.
- ϕ: the **angular component**. This is the angular distance of the point around the z-axis, moving counterclockwise from the x-axis, and it ranges from 0 to 2π.

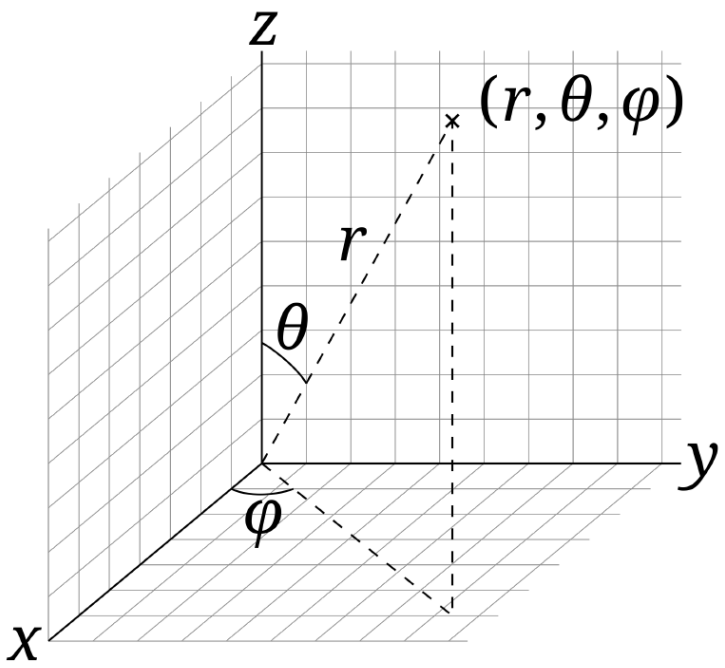


Figure 1.6. Diagram representing the components of the spherical coordinate system compared with Cartesian coordinates

If you have seen polar or cylindrical coordinates before, the definitions of θ and ϕ above might seem backwards. Furthermore, mathematicians use θ and ϕ the other way around, the opposite convention from physicists. This is an annoying and, at times, confusing inconsistency in notation, but we're stuck with it now ... In this course we will use the physicists' convention of θ being the azimuthal component and ϕ being the angular component.

Assuming the Earth were a perfect sphere, and that the origin of our spherical coordinate system were at the centre of the Earth, then:

- a point on the Earth's surface would have $r = R_{\text{Earth}}$. Points in the Earth's interior would have $r < R_{\text{Earth}}$, and points above sea level (or even in outer space) would have $r > R_{\text{Earth}}$.
- the North Pole would have $\theta = 0$, the South Pole would have $\theta = \pi = 180°$, and every point on the equator would have $\theta = \pi/2$.
- a point on the Prime Meridian (longitude 0°) would have $\phi = 0$. A point just east of it, would have a very small positive ϕ. A point just west of it would have ϕ just a bit smaller than 2π.

Using Figure 1.6 and some simple geometry, you can convert between Cartesian coordinates and spherical coordinates. Given a point $(t,\ \theta,\ \phi)$ in spherical coordinates, its position in Cartesian coordinates is given by:

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

> **2.** *Using your knowledge of trigonometry and Figure 1.6, verify that the equations above are in fact the correct way to convert from spherical coordinates to Cartesian coordinates!*

You can also go the other way, from Cartesian to spherical coordinates. Given a point in Cartesian coordinates, its position in spherical coordinates is given by:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\phi = \mathrm{sgn}(y)\cos^{-1}\left(\frac{x}{\sqrt{r^2 - z^2}}\right)$$

where sgn is the "sign function", which is -1 if its argument is negative, 1 if its argument is positive and 0 if its argument is 0.

1.2. Advanced Material (Optional)

This section is optional but contains some more advanced mathematics (no numbers — just concepts) that you may find useful for understanding the content in this module.

1.2.1. Waves on a String

When you pluck a string on a musical instrument, it makes a sound. What determines the sounds that the string produces? Why do a guitar, a banjo, and a violin sound different even when they are all playing the same note?

When both ends of a string are fixed and the string is pulled taut, the waves that can exist on the string are sinusoids with evenly spaced peaks and troughs, or equivalently, evenly spaced **nodes**, where a node is a point on the string, not counting the endpoints, that doesn't move up and down at all. Such a sinusoid can have a wavelength twice the length

of the string (a single peak, zero nodes), the length of the string (one peak and one trough, one node), two thirds the length of the string (two peaks and one trough, two nodes), half the length of the string (two peaks and two troughs, three nodes), etc. **(Fig. 1.7)**. In general a wave on a string will be a sinusoid with a wavelength equal to $\lambda_n = 2L/n$ and with $n - 1$ nodes, where n is a positive integer.

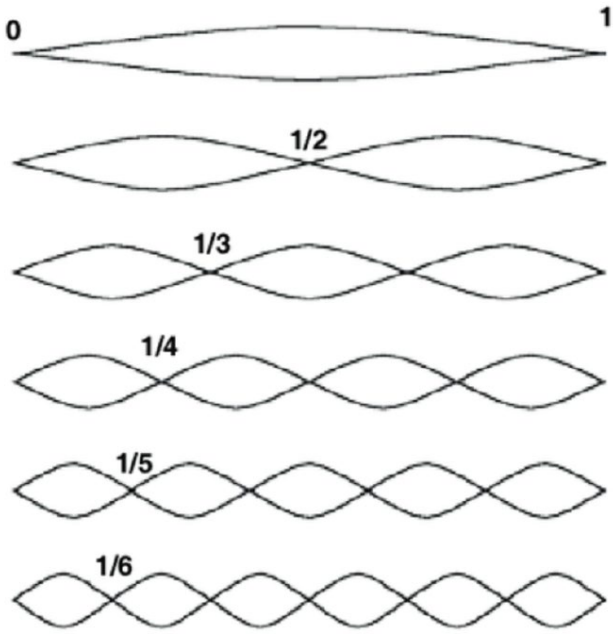


Figure 1.7. Different waves that are able to be formed on a string with fixed points at both ends of the string. Each string is represented twice, demonstrating the nodes on the string for different frequencies.

Also demonstrated above is the fact that waves with a higher spatial frequency (more nodes) also have a higher temporal frequency (they oscillate up and down more rapidly). This is because the temporal frequency of a wave depends on both the wavelength of the wave and the speed of sound in the string. This is why the note from a string becomes higher-pitcher as you tighten it — taut strings have a higher speed of sound than loose strings.

When you pluck a string, how does it "know" which of the allowed wavelengths should start playing? It doesn't — all (infinitely many) of them play simultaneously. The wave with the largest wavelength, $\lambda_1 = 2L$ — or equivalently, the lowest spatial (and temporal) frequency — is the fundamental tone. When you pluck the E string on an instrument, the sound of that E note is the wave whose wavelength is twice the length of the E string.

But all of the higher-frequency waves, known as overtones, also exist on the string. The physical properties of the string determine the relative amplitudes of all of the waves that you hear, which in turn determines what the string sounds like when you pluck it.

If you know the amplitudes of each wave on the string, then you completely know the behaviour of the string as it vibrates in space over time: simply evolve each component wave in time independently, then multiply them by their amplitudes and add up the results. The vertical displacement of the string at position x and time t would be written as the infinite sum:

$$y(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{2\pi x}{\lambda_n}\right) \cos(\omega_n t)$$

where A_n is the amplitude of the nth wave (determined by the physical properties of the string, as described above), and ω_n is the temporal frequency of the nth wave (determined by the wave's wavelength $\lambda_n 2L/n$ and the speed of sound in the string).

Since the vertical displacement of the string at each point in space and time is all of the information regarding the string, if we know the amplitudes of the individual waves on the string, and we know the medium the sound will travel in (air, water, *etc*.), then we know exactly how the string will sound when we pluck it.

1.2.2. Waves on a Sphere

Just as strings can support one-dimensional waves, spheres can support three-dimensional waves. Whereas a wave on a string is identified by just one number — the number of nodes it has along the length of the string — a wave on a sphere is determined by the number of nodes is has in the radial, azimuthal, and angular directions. And whereas a node on a string was simply a (zero-dimensional) point, nodes on a sphere are two-dimensional surfaces that cut through the sphere.

The vibrational mode of a wave is the triple (l, m, n), where l is the total number of nonradial nodes, m is the number of angular nodes (and so $l–m$ is the number of azimuthal nodes), and n is the number of radial nodes. Aside from the fact the m cannot exceed l, these are all independent from each other; a wave on a sphere can have any number of radial, angular, and azimuthal nodes.

In **Figure 1.8** is a visual of several vibrational modes on the surface of a sphere. A node is where the colour changes from a positive displacement from the surface of the sphere (blue) to a negative displacement (yellow) You can see the angular and azimuthal components on the surface of the sphere. The first column has $m = 0$, and each subsequent

column has one additional angular node. The first row has $l = 0$, and each subsequent row increases l by 1. The main diagonal is where $l - m = 0$ (where there are zero azimuthal nodes) and each subsequent diagonal has an additional azimuthal node. Not shown in this visualisation are the radial components of the waves. To see those, you'd have to cut into the sphere (and a third dimension would have to be added to the diagram)

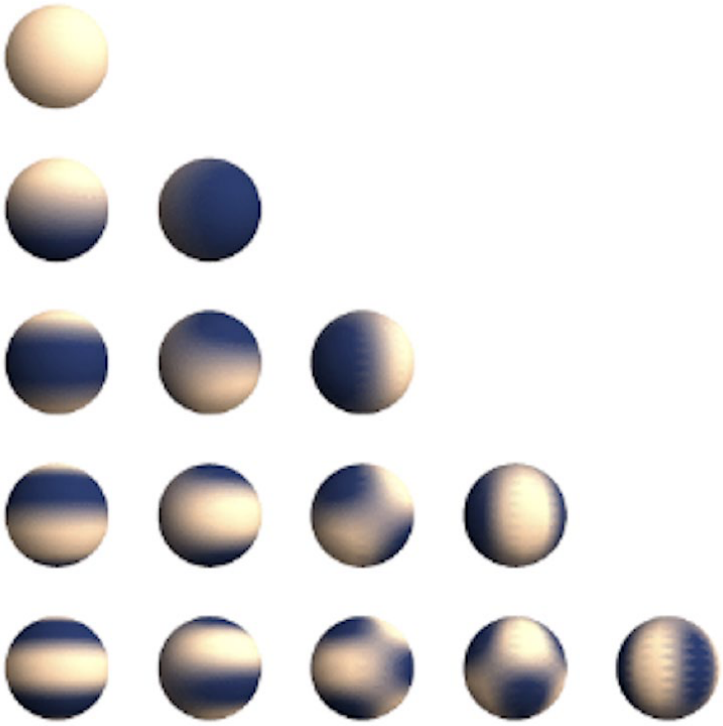


Figure 1.8. Different waves on a sphere. The blue areas represent where the surface is travelling away from the centre of the sphere, while the yellow areas represent where the surface is travelling towards the centre of the sphere. Where the blue and yellow areas meet, is a node.

Just as a plucked string will play all of the waves it can support simultaneously, a vibrating sphere (or star, in our case) will support all of the spherical harmonics it can support simultaneously. Furthermore, the physical properties of the sphere, such as its speed of sound, will determine the relationship between a given wave's wavelength and its temporal frequency.

1.2.3. Fourier Analysis

We've seen that a vibrating object will support (infinitely!) many different modes of vibration, and when you add up all of the vibrating waves multiplied by their amplitudes, you get the resulting wave form — how the vibrating medium sounds to the ear or appears to the eye.

Can we go the other direction? If we encounter an arbitrary waveform — if our ears hear a sound played over time, or if we use a microphone to record a sound to a computer — is it possible to determine the amplitudes of the individual sinusoidal waves that made up the vibrating medium that we heard? If we observe how a sphere appears to vibrate, can we determine the amplitudes of the vibrational modes that comprise the sphere's vibration?

Yes, through Fourier analysis. Fourier analysis takes a signal that you have recorded over time, like the plot of amplitude with respect to time shown in **Figure 1.9**, a recording of the sound pressure created by a bass guitar string, and tells you the amplitude of each frequency component, as shown in **Figure 1.10**, which gives the amplitude with respect to frequency. We say that the Fourier transform has been performed between the time domain (Fig. 1.9) and the frequency domain (Fig. 1.10).

Figure 1.9 — which is a graph of the pressure waves hitting our ears or the microphone — doesn't tell you much about the sound, except that it is periodic. However, Figure 1.10 tells you exactly which frequencies comprise this sound hitting our ears.

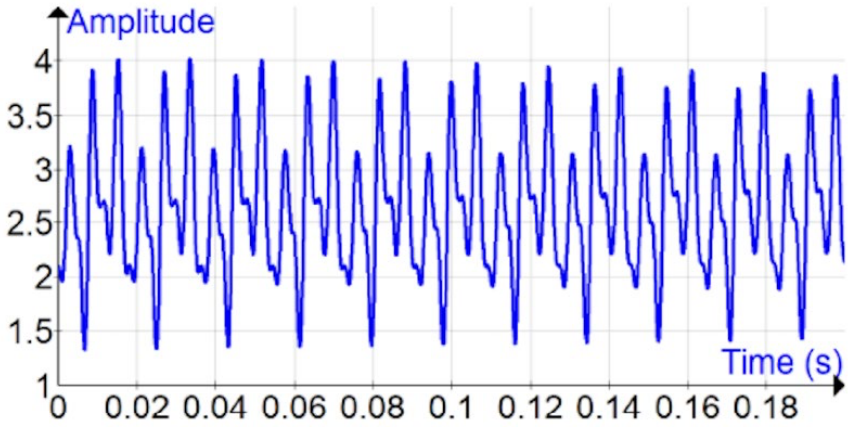


Figure 1.9. A plot of amplitude against time for a bass guitar string, showing the variation in amplitude of the wave

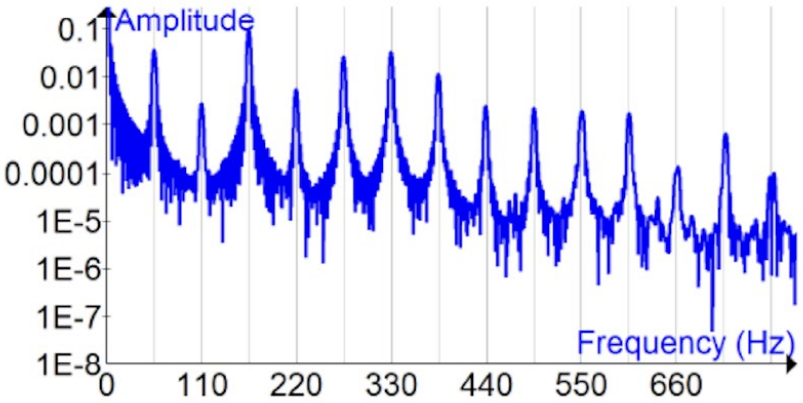


Figure 1.10. A plot of the Fourier transform of Figure 9, showing the amplitude against the frequency. Note here that the amplitude in this plot is not the amplitude of the wave, but a measurement of how much each frequency contributes to the sound that you hear.

Can you see the fundamental frequency and overtones of this string? What musical note does this string on the bass guitar correspond to? (The peak at 0 Hz is simply the average amplitude over all time, which doesn't represent an actual wave. If you shifted the time-domain graph so that it was centred at $A = 0$, the 0 Hz peak would disappear, whereas the other peaks would remain.)

This is what Fourier analysis is all about: by observing the amplitude of some signal over time, we can determine the amplitudes of the underlying frequencies that generated that signal. How this is actually achieved mathematically is beyond the scope of this course.

By the way: this is how our ears work: our inner ears have cells with tiny hair-like structures that each vibrate in response to a different frequency in the incoming sound, and they're connected to nerves that tell our brains how much each hair is vibrating. So our ears actually perform a Fourier transform on incoming sound and report the results to our brains, which then reconstitutes the result into what we perceive as sound.

You should now have enough mathematical background to tackle the material in this module. The study of asteroseismology is about applying the maths above to the stars we actually observe.

You can find the theory videos here:
www.youtube.com/playlist?list=PLrGAIWCGSKlA_CP3hLKbbdtRaYT33Aaqa

2. Exercises for Chapter 1

As explained in the Chapter 1 video, there are three important time scales for stellar evolution: the hydrostatic time scale, the contraction time scale, and the nuclear time scale:

$$\tau_{\rm hydro} \approx \sqrt{\left(\frac{R^3}{GM}\right)} \qquad \tau_{\rm HK} \approx \frac{GM^2}{2RL} \qquad \tau_{\rm n} \approx \frac{E_n}{L}$$

where M, R, L are the mass, radius, and luminosity of the star, respectively. Further, G is the gravitational constant (see Appendix 1 for its value) and $E_{\rm n}$ represents the energy reservoir built up by nuclear reactions. The main reactions during the largest fraction of the stellar life are those that achieve the fusion of four ^{1}H nuclei in one ^{4}He nucleus. This **core hydrogen burning** releases an energy corresponding to a **mass deficit** of about 0.75%. This is equal to the total mass of four protons minus the mass of a helium nucleus, divided by the total mass of four protons. The mass deficit per fusion reaction from 4 protons to a helium isotope creates an energy given by $(4m_{\rm p})\cdot(0.75\%)\cdot c^2$, with c the speed of light and $m_{\rm p}$ the mass of a proton (given in the **Appendix 1**). Hence, during the core hydrogen burning stage of a star, about 0.75% of its initial hydrogen mass fraction gets transformed into energy in the nuclear reactor situated in its deep interior. In practice, you can assume that about 10% of the star's birth mass are hydrogen isotopes situated in the inner region of the star where they are hot and dense enough to take part in the nuclear fusion.

EXERCISE 1 In the first part of this exercise, your aim is to compute the three time scales for six stars in the core hydrogen burning stage of their evolution, having the following birth masses:

(a) the birth mass of the Sun (denoted as 1.0 $M_\odot$)

(b) 0.5 $M_\odot$

(c) 1.5 $M_\odot$

(d) 2.0 $M_\odot$

(e) 10 $M_\odot$

(f) 30 $M_\odot$

In order to achieve that, you need to estimate the radius and luminosity of these stars. You can do that by relying on the so-called **mass-luminosity and mass-radius relations**. Indeed, it turns out that stars in the core hydrogen burning phase adhere well to these two relations:

$$L \sim M^{\eta} \qquad\qquad R \sim M^{\xi}$$

where the symbol "$\sim$" means "is proportional to".

Numerical computer models of stars result in $\eta \approx 3.35$, while $\xi \approx 0.81$. These two approximations are shown in **Figure 2.1** by the dashed lines. The figure shows the outcome of numerical computer models for star of different masses in the core hydrogen burning stage as a full line.

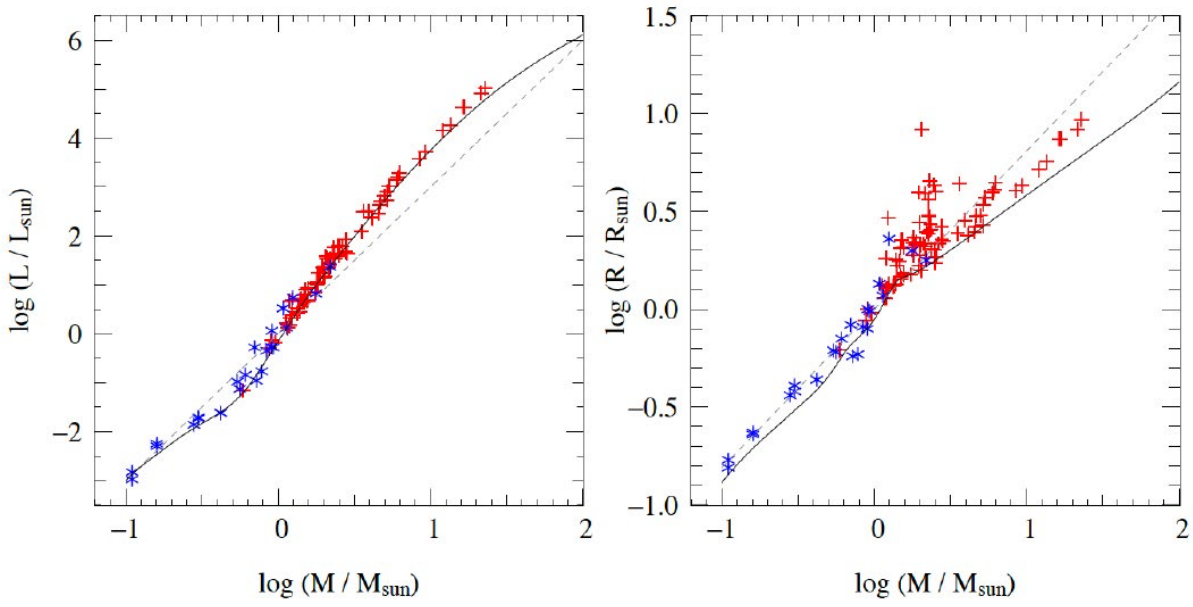


Figure 2.1. Mass-luminosity (left) and mass-radius (right) relation derived from numerical computer models mimicking stars with chemical composition similar to the Sun. When astrophysicists write "log", they actually mean "$\log_{10}$". The dashed lines show the approximations for the M-L and M-R relations with $\eta \approx 3.35$ (left panel) and $\xi \approx 0.81$ (right panel). The coloured symbols are masses and radii deduced from observations of binary stars, which are deduced from their orbital motion without the use of stellar models. The accuracies of these binary measurements are smaller than the symbol sizes. Blue stars: visual binaries; red plusses: spectroscopic binaries. (Figure courtesy of Prof. Onno Pols, Lecture Notes on Stellar Structure and Evolution, Radboud University Nijmegen, The Netherlands)

The exercise is easiest to solve by relying on the relations $L \sim M^{3.35}$ and $R \sim M^{0.81}$, where you work in solar units, but you can of course stick to SI units if you prefer it (you will have to deal with big numbers in that case!). **Appendix 1** of this document gives you the rele-

vant numerical values to solve this exercise, including the mass, radius, and luminosity values of the Sun.

EXERCISE 2 In the Chapter 1 video, the lecturer claimed that $\tau_{\rm n} >> \tau_{\rm HK} >> \tau_{\rm hydro}$ for all stars with core hydrogen burning. As a scientist, it is always a good attitude to be critical about what lecturers tell you. Hence, in this second part of the exercise you should check whether this relationship is indeed correct for all six stars. Try to decide for yourself how good/bad the approximation is for these stars.

EXERCISE 3 The nuclear time scale $\tau_{\rm n}$ is a good approximation for the total life time of a star, T. It corresponds to about 90% of the overall age that the star will reach when it dies. Use this information to calculate — more accurately than mentioned by the lecturer in the Chapter 1 video — how many generations of 30 $M_\odot$ stars will have died during the entire life time of the Sun. And how many generations of 10 $M_\odot$ stars?

Some reflections connected to this exercise: All the stars born with a mass above some 8 $M_\odot$ will create materials up to iron and nickel in the chemical periodic table of Mendeleyev during nuclear fusion in their inner core region, before exploding as supernova. During that explosion, complex nuclear reactions take place and make even heavier chemical elements, as shown in the chemical table of elements found in the solar system used by astrophysicists and illustrated on a slide in Chapter 1 by the lecturer. These massive stars give all the materials that they created in their interior during their life back to the interstellar medium of their home galaxy. In this way, this interstellar medium gets enriched with heavy chemical elements from which new stars will get born, so the more generations of massive stars have died, the more enriched the new stars will be. That is why this third exercise is important for astrophysics as it determines the chemistry of the evolving Universe. Cecilia Payne **(Fig. 2.2)** was the first astrophysicist who understood how this works. Thanks to this exercise, you will now understand why stars born with $M > 8\,M_\odot$ are often called "the chemical factories of the Universe".



Figure 2.2. Cecilia Payne-Gaposchkin (1900–1979) at Harvard College Observatory (Photograph by Smithsonian Institution / Science Service, restored by Adam Cuerden, Air and Space Museum online gallery, Public Domain)

The Chapter 2 and 3 videos gave you an introduction into stellar oscillations. Irrespective of how oscillations are excited in stars, they can be considered as *small perturbations* disturbing the equilibrium state of a star. You have now calculated the hydrostatic time scale of six different stars while they burn hydrogen in their core. This time scale expresses how long the star needs to recover its equilibrium state due to small disturbances. So now you learned what type of values to expect for the oscillation periods in order for the oscillations to be a sustainable phenomenon for each of these stars. Indeed, oscillations with periods longer than τ_{hydro} will damp out as the star will manage to return to equilibrium before the oscillation cycle is finished.

3. Project: Let's Make You a Real Asteroseismologist!

For the project work of the course, we distil some aspects from the Chapters 2 and 3 videos that are manageable to do by high-school students, with the aim to turn you into an asteroseismologist. Each of you will be analysing your own star observed by the NASA Kepler mission, whose power density spectrum is made available to you as an image file with name `StarXX.png`. All these stars have the effective temperature of their atmosphere, denoted as $T_{\rm eff}$, measured from spectroscopy according to **Appendix 2**. Each star reveals solar-like oscillations resembling those of the Sun. The helioseismic power density spectrum is shown in **Figure 3.1** for you as a guideline to help you with your research.

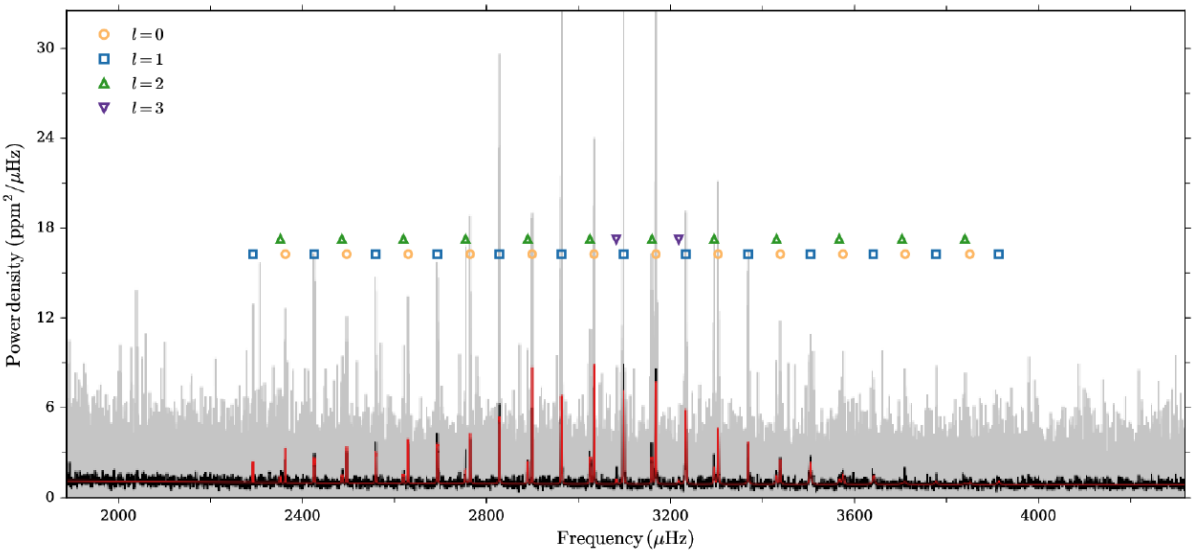


Figure 3.1. The power density spectrum of the Sun analysed as if it were a distant star, is shown, as a function of the frequency, by the grey curve. The black curve is a smoothed version of the grey curve obtained by computing a particular type of average (whose details do not matter, but it was obtained by means of a sliding window with a box width of 1 μHz, with background signal from other phenomena such as granulation subtracted). Theoretical predictions for the solar oscillations computed from an equilibrium model of the Sun are overlaid (red curve). This theoretical model was used to identify the solar oscillations. The markers indicate the degree l of the identified modes at their frequency value. You can use this graph to identify the degrees l of the oscillation modes of your own star.

(a) As a first part of the project, try to identify the degree l of the dominant oscillation modes of your own star, using Figure 3.1 as a guide. You can do that from the grey power spectrum of your star. The more modes you can identify securely the better, but do avoid to over-interpret doubtful frequency peaks that do not belong to a regular oscillation pattern (we do not wish to interpret noise peaks as if they are physically real, nor frequencies due to other phenomena than oscillations!).

(b) Deduce the frequency of maximum power, $\nu_{\rm max}$, from the power density spectrum. Compare it to the value for the Sun: $\nu_{\rm max,\odot} = 3100 \pm 30$ μHz. Is your star larger or smaller than the Sun?

(c) Compute an average for the large frequency separation, $\langle \Delta \nu_{\rm nl} \rangle$, from the power density spectrum. The large frequency spacing can be deduced from the approximate formula

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \epsilon \right) \Delta \nu_{nl}$$

with ϵ a constant (whose value is of no importance for the exercise). Hence, if you subtract frequencies of two modes of the same degree l and whose radial orders (n) differ by unity (so-called modes of consecutive radial order), you can estimate $\Delta \nu_{\rm nl}$. Try to estimate $\Delta \nu_{\rm nl}$ as well as you can, and also its uncertainty. Compare it to the value for the Sun deduced by helioseismologists: $\langle \Delta \nu_{\rm nl} \rangle = 134.9 \pm 0.1$ μHz. There are various ways you can compute $\langle \Delta \nu_{\rm nl} \rangle$, but all of them require you to distil the frequency values for the low-degree modes (of $l = 0,\ 1$ or 2; sometimes $l = 3$ modes are also detectable but not for every star) of consecutive radial orders n. So you have to rely on the numerical frequency values in the data file for the (best cases of the) modes whose degree l you managed to identify unambiguously.

(d) Use $\langle \Delta \nu_{\rm nl} \rangle$, $\nu_{\rm max}$, and the scaling relations explained in the Chapter 2 video to deduce the mass and radius of your star, as well as their uncertainties, assuming that the star's chemical composition at birth was not too different from the Sun's composition. Rely on the measurement of the effective temperature of your star deduced from spectroscopy as listed in Appendix 2.

(e) Try to deduce an average value for the small frequency separation from radial ($l = 0$) and quadrupole ($l = 2$) modes, defined as $\delta \nu_{02} \equiv \nu_{\rm n,l=0} - \nu_{\rm n-1,l=2}$. Use the figure for the Sun discussed and shown in the Chapter 2 video and apply it to your star from its identified radial and quadrupole modes. Try also to estimate its uncertainty. The mathematical integral expressions giving the physical meaning of the large and small frequency separations were presented in the slides discussed by the lecturer in the Chapter 2 video, for the Sun and for the Sun-like star 16 Cyg. You have now deduced measured values that you can interpret astrophysically. To help you with that, we provide **Figure 3.2**. For the Sun, the value determined by helioseismologists is $\langle \delta \nu_{02} \rangle \approx 9$ μHz, leading

to a helioseismic age of $\tau \approx 4.58 \pm 0.02$ Gyr. Place the Sun on the asteroseismic age-dating diagram for low-mass stars shown in Figure 3.2. Place your star there as well. Is your star older or younger than the Sun? Why?

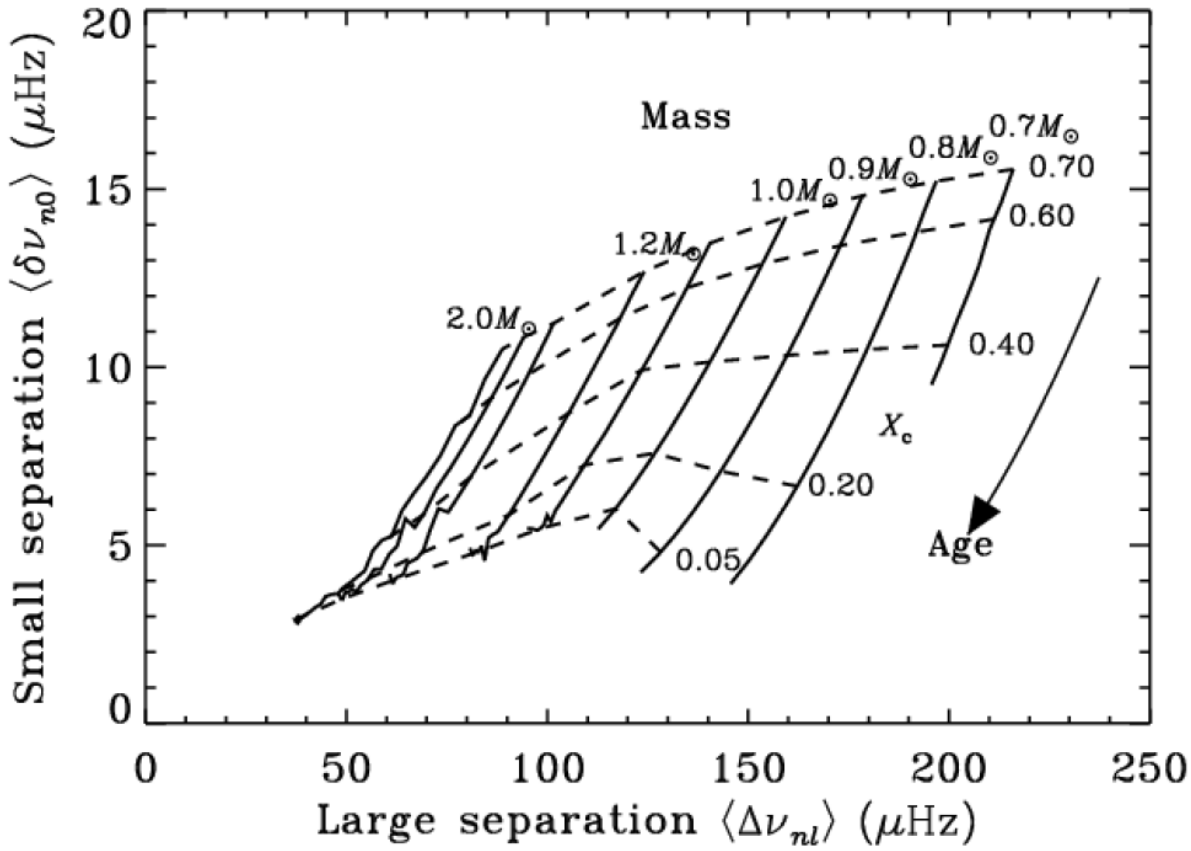


Figure 3.2. A helioseismic age-dating diagram showing the small versus the large frequency separation of evolving stellar models in the core hydrogen burning stage (full black lines) with birth masses between 0.7 (upper right) and 2.0 $M_\odot$ (leftmost black line). All these computer models have the same initial chemical composition as the Sun. Stars are born somewhere on the upper dashed line where their central hydrogen mass fraction (denoted as X_c) equals 0.70 as labelled. As the stars grow older, they move downward along a black line in this diagram towards lower values of X_c. The end of the core hydrogen burning stage has $X_c = 0.0$. (Figure courtesy by Prof. Jorgen Christensen-Dalsgaard, Aarhus University, Denmark)

4. Exercise for Chapter 4

Liquid water can exist in a temperature regime of about 0 to 100 °C, which corresponds to a range from 273 to 373 K. In this exercise you will try to answer the question which of the currently known rocky exoplanets whose mass and radius could be determined accurately so far are situated in the habitable zone of their host star. The masses of the host stars with detected Earth-like rocky exoplanets is plotted with respect to their exoplanet orbital distances in **Figure 4.1**. Here the data are shown for all known cases of rocky exoplanets with radii smaller than twice the size of Earth and masses known to better than 20% precision.

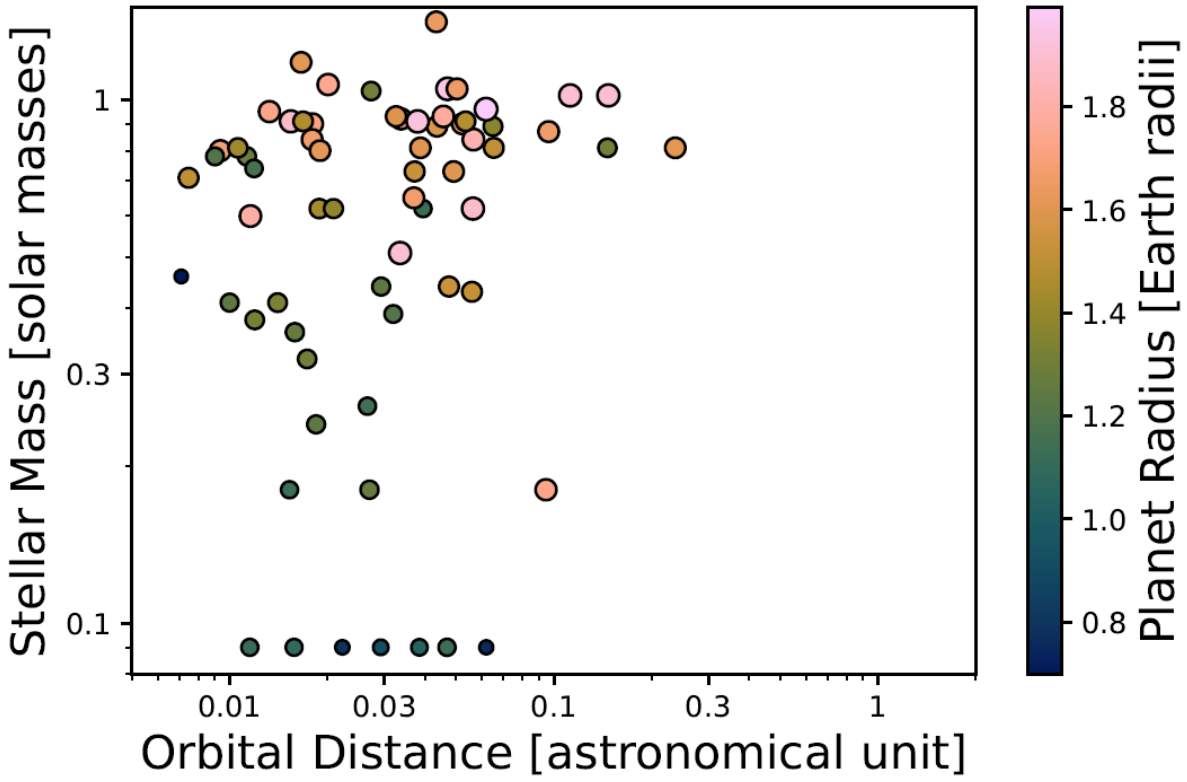


Figure 4.1. Mass of the host star versus orbital distance of its rocky exoplanets (semi major axis of the orbit, expressed in AU) for all known cases with radii smaller than twice the size of Earth and masses known to better than 20% precision (source: NASA exoplanet archive). The symbols are colour-coded and sized according to the exoplanet's radius.

To a good approximation, the temperature of the exoplanet, T_{planet}, at a distance D away from the host star can be approximated as follows:

$$T_{\text{planet}} \approx A \cdot \left(\frac{L}{16\pi\sigma D^2}\right)^{1/4}$$

where σ is the constant of Stefan-Boltzmann (see **Appendix 1**) and A is a dimensionless factor depending on the reflection and absorption properties of the planet including the planet's surface and atmosphere.

(a) Use this approximation to:

- **i.** Estimate A from the average temperature here on Earth, which is 288 K. The Earth is situated at 1 Astronomical Unit (1 AU) from the Sun. Mind you: you are calculating a temperature that has to have the physical unit of degrees Kelvin, so be careful not to make a mistake with all the units of the parameters that enter the formula!
- **ii.** Compute the habitable zone, *i.e.* the zone for which $T_{\text{planet}} \in [273, 373]$ K, for host stars covering the mass range shown in Figure 4.1, where you assume the absorption and reflection properties of all these planets are the same as those of the Earth (i.e., use the A value that you obtained in part i of the exercise). Some aspects of Exercise 1 will come in handy to solve the current exercise!

(b) Draw (by hand) the habitable zone as a region for all the host stars in Figure 4.1 (a rough sketch will do fine). How many of the currently known rocky exoplanets shown in Figure 4.1 are in the habitable zone of their host star and are worth searching for extraterrestrial life? In your opinion, is the pool of already known exoplanets with accurate mass and in the habitable zone of their host star large enough to search for extraterrestrial life, or do we need to discover more of them? Why?

Appendix 1: Physical and Astronomical Constants

Some physical constants (you do not need all of these for the exercises):

Constant (symbol)	CGS units	SI units
Speed of light (c)	$2.99792458 \times 10^{10}$ cm·s^{-1}	2.99792458×10^{8} m·s^{-1}
Gravitation (G)	6.67259×10^{-8} cm^3·g^{-1}·s^{-2}	6.67259×10^{-11} m^3·kg^{-1}·s^{-2}
Atomic Mass Unit (m_u)	$1.6605390 \times 10^{-24}$ g	$1.6605390 \times 10^{-27}$ kg
Mass electron (m_e)	$9.1093836 \times 10^{-28}$ g	$9.1093836 \times 10^{-31}$ kg
Mass proton (m_p)	$1.6726219 \times 10^{-24}$ g	$1.6726219 \times 10^{-27}$ kg
Mass neutron (m_n)	$1.6749275 \times 10^{-24}$ g	$1.6749275 \times 10^{-27}$ kg
Mass helium nucleus (m_{He})	$6.6446572 \times 10^{-24}$ g	$6.6446572 \times 10^{-27}$ kg
Charge electron (e)	$1.60217733 \times 10^{-20}$ c esu	$1.60217733 \times 10^{-19}$ C
Planck ($h = 2\pi\hbar$)	$6.6260755 \times 10^{-27}$ erg·s	$6.6260755 \times 10^{-34}$ J·s
Boltzmann (k)	1.380658×10^{-16} erg·K^{-1}	1.380658×10^{-23} J·K^{-1}
Radiation (a)	7.5646×10^{-15} erg·cm^{-3}·K^{-4}	7.5646×10^{-16} J·m^{-3}·K^{-4}
Stefan-Boltzmann (σ)	5.67051×10^{-5} erg·cm^{-2}·s^{-1}·K^{-4}	5.67051×10^{-8} J·m^{-2}·s^{-1}·K^{-4}

Some astrophysical constants:

Constant (symbol)	CGS units	SI units
Radius Sun ($R_\odot$)	6.9598×10^{10} cm	6.9598×10^{8} m
Mass Sun ($M_\odot$)	1.9891×10^{33} g	1.9891×10^{30} kg
Luminosity Sun ($L_\odot$)	3.8515×10^{33} erg·s^{-1}	3.8515×10^{26} J·s^{-1}
Astronomical Unit (AU)	1.49598×10^{13} cm	1.49598×10^{11} m
Light year (ly)	9.463×10^{17} cm	9.463×10^{15} m

Transformations:

From Ångström to cm:	1 Å = 10^{-8} cm
From Newton to dyne:	1 N = 10^{5} dyne
From Joule to erg:	1 J = 10^{7} erg
From electronvolt to erg:	1 eV = $1.60217733 \times 10^{-12}$ erg
From atmosphere to dyne·cm^{-2}:	1 atm = 1.01325×10^{6} dyne·cm^{-2}

Appendix 2: Effective Temperatures of Stars

File Name	T_{eff} (K)	File Name	T_{eff} (K)	File Name	T_{eff} (K)
Star01.png	6326 ± 77	Star22.png	5775 ± 77	Star43.png	6538 ± 77
Star02.png	6614 ± 77	Star23.png	6171 ± 77	Star44.png	6278 ± 77
Star03.png	6045 ± 77	Star24.png	5811 ± 77	Star45.png	6047 ± 77
Star04.png	6384± 77	Star25.png	6248 ± 77	Star46.png	6253 ± 75
Star05.png	6193 ± 77	Star26.png	5501 ± 77	Star47.png	6321 ± 77
Star06.png	5668 ± 77	Star27.png	6235 ± 77	Star48.png	5457 ± 77
Star07.png	6107 ± 77	Star28.png	5309 ± 77	Star49.png	5860 ± 180
Star08.png	5805 ± 77	Star29.png	5488 ± 77	Star50.png	6132 ± 77
Star09.png	5846 ± 77	Star30.png	6173 ± 101	Star51.png	5949 ± 77
Star10.png	6130 ± 84	Star31.png	6343 ± 77	Star52.png	6146 ± 77
Star11.png	5853 ± 77	Star32.png	6122 ± 77	Star53.png	6177 ± 77
Star12.png	6037 ± 77	Star33.png	6067 ± 120	Star54.png	5964 ± 77
Star13.png	6033 ± 77	Star34.png	6143 ± 77	Star55.png	6045 ± 77
Star14.png	6313 ± 76	Star35.png	5719 ± 77	Star56.png	6150 ± 180
Star15.png	6331 ± 77	Star36.png	6246 ± 77	Star57.png	6140 ± 77
Star16.png	5674 ± 77	Star37.png	5873 ± 77	Star58.png	6548 ± 77
Star17.png	6479 ± 77	Star38.png	5677 ± 77	Star59.png	6642 ± 77
Star18.png	5832 ± 77	Star39.png	5270 ± 180	Star60.png	6179 ± 77
Star19.png	6344± 77	Star40.png	5852 ± 77	Star61.png	6276 ± 77
Star20.png	6068 ± 102	Star41.png	6302 ± 77	Star62.png	5825 ± 50
Star21.png	6305 ± 77	Star42.png	6400 ± 84	Star63.png	5750 ± 50